Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$$a = \frac{bm}{\sqrt{b^2 - n^2}}.$$

The area of the ellipse, which is easily found by integration, is πab. Substituting the above value of a in this expression we have

$$\frac{\pi b^2 m}{\sqrt{b^2 - n^2}},$$

which is a function, $f(b)$, of the independent variable b. To find the minimum value of $f(b)$ find the values of b which cause the first derivative to vanish.

$$f'(b) = \pi m \frac{2b(b^2 - n^2) - b^3}{(b^2 - n^2)^{3/2}} = 0.$$

Solving this equation we find b to be 0 or $n\sqrt{2}$. The latter value is obviously the desired one. The corresponding value of a is $m\sqrt{2}$.

Hence, the area is $\pi m\sqrt{2} \cdot n\sqrt{2}$ or $2\pi mn$.

From geometrical considerations it does not seem necessary to show that $f''(b)$ is positive and that, therefore, $f(b)$ is a true minimum.

Also solved similarly by H. S. UHLER, H. C. FEEMSTER, GEORGE W. HARTWELL, and H. L. AGARD.

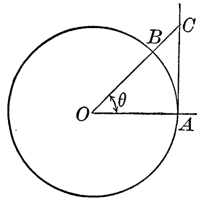
Note.—The above solution assumes that the bricks are laid side by side without mortar, whereas in a "brick wall" they are laid so as to *break joints*. Very possibly the question as answered is the one really intended by the proposer but the ellipse found certainly does not inclose four bricks as they are laid in a *brick wall*. EDITORS.

394. Proposed by W. W. BURTON, Macon, Ga.

A horse runs 10 miles per hour on a circular race-track in the center of which is an arc-light. How fast will his shadow move along a straight board fence (tangent to the track at the starting point) when he has completed one eighth of the circuit?

SOLUTION BY C. E. HORNE, Westminster College, Colorado.

Let B and C be the position of the horse and shadow at any time after starting from the point of tangency, A. Let $AB = s$, $AC = y$, angle $AOC = \theta$, and $r =$ the radius of the ring. Then $s = r\theta$ (1) and $y = r \tan \theta$ (2). From (1), $ds/dt = r(d\theta/dt)$, the rate of the horse and from (2), $dy/dt = r \sec^2 \theta(d\theta/dt)$, the rate of the shadow, $= \sec^2 \theta(ds/dt) = 10 \sec^2 \theta = 20$ miles per hour when $\theta = \pi/4$.



Also solved by A. H. HOLMES, H. L. AGARD, J. A. CAPARO, D. RUMBLE, W. C. EELLS, CLIFFORD N. MILLS, HORACE OLSON, H. L. AGARD, and GEORGE W. HARTWELL.

395. Proposed by W. W. BURTON, Mercer University, Macon, Ga.

Into a full conical wine glass whose depth is a and whose angle at the base is 2α there is carefully dropped a spherical ball of such size as to cause the greatest overflow. Show that the radius of the ball is $a \sin \alpha/(\sin \alpha + \cos 2\alpha)$.

From Woods and Bailey's *A Course in Mathematics* (1907), Volume I, page 213.

SOLUTION BY H. S. UHLER, Yale University.

Let the sphere touch the inside of the cone near the rim with its center above the plane of the edge. Also, let h and r denote, respectively, the altitude of the submerged spherical segment and the radius of the sphere. Then $h = a + r(1 - \csc \alpha)$. The volume v of the liquid spilled is, of course, equal to the volume of the submerged spherical segment.

The volume of a spherical segment of altitude h, and radius of base r_1 is $\frac{1}{6}\pi h^3 + \frac{1}{2}\pi h r_1^2$. Here,

$$r_1 = \sqrt{2rh - h^2}, \qquad \text{hence} \qquad v = \pi h^2(r - \tfrac{1}{3}h).$$

Now,

$$\frac{dv}{dr} = \pi h\left[h + (2r - h)\frac{dh}{dr}\right], \qquad \text{and} \qquad \frac{dh}{dr} = 1 - \csc\alpha.$$

Therefore,

$$\frac{dv}{dr} = \pi h[a\csc\alpha - r(\csc\alpha - 1)(\csc\alpha + 2)].$$

Obviously, $h = 0$ cannot give a maximum overflow. Consequently,

$$r = \frac{a\csc\alpha}{(\csc\alpha - 1)(\csc\alpha + 2)} = \frac{a\sin\alpha}{(1 - \sin\alpha)(1 + 2\sin\alpha)} = a\sin\alpha/(\sin\alpha + \cos 2\alpha).$$

Furthermore,

$$\frac{d^2v}{dr^2} = -\pi h(\csc\alpha - 1)(\csc\alpha + 2).$$

Since $h \neq 0$ and $\csc\alpha > 1$, d^2v/dr^2 is negative so that the condition for a maximum is satisfied.

Also solved by A. H. Wilson, S. E. Rasor, Elmer Schuyler, George Raynor, C. N. Schmall, H. L. Agard, C. N. Mills, Lewis Clark, L. M. Pickett, C. A. Bergstresser, C. Hornung, L. G. Weld, J. A. Bullard, G. W. Hartwell, J. V. Balch, J. A. Whitted, Elijah Swift, J. C. Rayworth, and J. A. Caparo.

396. Proposed by ELBERT H. CLARKE, Purdue University.

The length of the curve $y = x^n$ from the origin to the point (1, 1) is given by the formula

$$= \int_0^1 \sqrt{1 + n^2x^{2n-2}}\cdot dx.$$

Our geometric intuition would tell us that the limit of this length as n becomes infinite is 2. Give a strict analytic proof that

$$\lim_{n\to\infty} \int_0^1 \sqrt{1 + n^2x^{2n-2}}\cdot dx = 2.$$

Solution by Elijah Swift, University of Vermont.

We can easily show that

(1)
$$\lim_{n\to\infty} n^2x^{2n-2} = 0, \qquad 0 \leqq x < 1.$$

Next, we choose a positive number, ϵ, as small as we like. We can then find a value N of n such that for it and all larger values

(2)
$$n^2(1 - \epsilon)^{2n-2} < \eta,$$

where η is a positive number as small as we like. This is possible on account of (1).

Let x_n be a value of x such that $n^2{x_n}^{2n-2} = 2$.

Now write the given integral, I, as a sum of three integrals.

$$I = \int_0^{1-\epsilon} \sqrt{1 + n^2x^{2n-2}}\cdot dx + \int_{1-\epsilon}^{x_n} \sqrt{1 + n^2x^{2n-2}}\cdot dx + \int_{x_n}^{1} \sqrt{1 + n^2x^{2n-2}}\cdot dx.$$

Considering values of $n > N$, we have

(3)
$$\begin{aligned}\left|\int_0^{1-\epsilon} \sqrt{1 + n^2x^{2n-2}}\cdot dx - 1\right| &= \left|\int_0^{1-\epsilon}\left\{\sqrt{1 + n^2x^{2n-2}} - 1\right\}dx - \epsilon\right| \\ &\leqq \left|\int_0^{1-\epsilon}\left\{\sqrt{1 + \eta} - 1\right\}dx\right| + \epsilon < \frac{\eta}{2} + \epsilon. \quad \text{(See (2).)}\end{aligned}$$

This holds if $n \geqq N$.

Now

(4)
$$\int_{1-\epsilon}^{x_n} \sqrt{1 + n^2x^{2n-2}}\cdot dx < \int_{1-\epsilon}^{1} \sqrt{1 + 2}\cdot dx = \sqrt{3}\cdot\epsilon.$$